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Almost-commutative geometries beyond the standard model II: new colours

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Abstract

We will present an extension of the standard model of particle physics in its almost-commutative formulation. This extension is guided by the minimal approach to almost-commutative geometries employed by Iochum *et al* (2004 *J. Math. Phys.* **45** 5003 (*Preprint hep-th/0312276*)), Jureit and Stephan (2005 *J. Math. Phys.* **46** 043512 (*Preprint hep-th/0501134*)), Schücker (2005 *Preprint hep-th/0501181*), Jureit *et al* (2005 *J. Math. Phys.* **46** 072303 (*Preprint hep-th/0503190*)) and Jureit and Stephan (2006 *Preprint hep-th/0610040*), although the model presented here is not minimal itself. The corresponding almost-commutative geometry leads to a Yang–Mills–Higgs model which consists of the standard model and two new fermions of opposite electromagnetic charge which may possess a new colour-like gauge group. As a new phenomenon, grand unification is no longer required by the spectral action.

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1. Introduction

Understanding the origin of the standard model is currently one of the most challenging issues in high-energy physics. Indeed, despite its experimental successes, it is fair to say that its structure remains a mystery. Moreover, a better understanding of its structure would provide us with a precious clue towards its possible extensions. This can be achieved in the framework of noncommutative geometry [1], which is a branch of mathematics pioneered by Alain Connes, aiming at a generalization of geometrical ideas to spaces whose coordinates fail to commute. Motivated by quantum gravity, it is postulated that spacetime is a wildly noncommutative

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manifold at a very high energy. Even if the precise nature of this noncommutative manifold remains unknown, it seems legitimate to assume that at an intermediate scale, say a few orders of magnitude below the Planck scale, the corresponding algebra of coordinates is only a mildly noncommutative algebra of matrix-valued functions, called almost-commutative geometries. When suitably chosen, such a matrix algebra, a sum of three simple matrix algebras, reproduces, within the spectral action principle, the standard model coupled to gravity [2].

Ten years after its discovery [2], the spectral action has recently received new impetus [3–5] by allowing a Lorentzian signature in the internal space. This mild modification has three consequences. The fermion-doubling problem [6] is solved elegantly, Majorana masses and consequently the popular seesaw mechanism are allowed for. The Majorana masses in turn decouple the Planck mass from the W mass. Furthermore, Chamseddine, Connes and Marcolli point out an additional constraint on the coupling constants tying the sum of all Yukawa couplings squared to the weak gauge coupling squared. This relation already holds for Euclidean internal spaces [7].

For many years, it has been tried to construct models from noncommutative geometry that go beyond the standard model [8]. But these attempts failed to come up with anything physical if it was not to add more generations and right-handed neutrinos to the standard model. For example, the noncommutative constraints on the continuous parameters of the standard model with four generations fail to be compatible with the hypothesis of the big desert [9].

The situation changed recently, when a classification of the finite part of almost-commutative geometries with up to four summands in the matrix algebra was performed [13]. This classification necessitated the heavy use of a computer program [10] to list the irreducible Krajewski diagrams. Here, the standard model appears in a most prominent position. But also the so-called electro-strong model was discovered which inspired the first viable almost-commutative model beyond the standard model: the AC -model [11]. It comes from an algebra with six summands and is identical to the standard model with two additional leptons A^{2-} and C^{2+} whose electric charge is 2 in units of the electron charge. These new leptons couple neither to the charged gauge bosons, nor to the Higgs scalar. Their hypercharges are vector-like, so that they do not contribute to the electro-weak gauge anomalies. Their masses are gauge-invariant and they constitute viable candidates for cold dark matter [12].

In this paper, we will use a version of the standard model based on a matrix algebra with four summands [13]. We will investigate a new extension of the standard model which is also inspired by the classification of irreducible almost-commutative geometries [13] and extends the standard model by N generations of left-handed $SU(2)$ doublets and right-handed singlets. These new particles may also possess a new colour group, $SU(D)$, with respect to which the standard model particles are singlets, i.e. neutral. They resemble closely to the θ particles which were proposed by Okun [14].

One of the main results of this paper is the fact that the constraints on the gauge couplings of the standard model no longer resemble those of grand unified theories. The new relation is given in equation (4.12). If the extensions of the standard model interact via the weak or the strong interactions, this seems to be a general feature in almost-commutative geometry. For colour groups $SU(D)$ with $D \geq 3$, one finds also that at least three generations of new particles are needed.

The paper is organized as follows. We first give the basic notions of a spectral triple, the main building block of noncommutative geometry. Then we quickly review how the Yang–Mills–Higgs model is obtained via the fluctuated Dirac operator and the spectral action. This account is far from exhaustive and we refer to [2, 5, 15] for a detailed presentation.

For the new particles, the details of the spectral triple and the lift of the automorphisms are given. The Lagrangian as well as the constraints on the couplings are calculated. With the help of the one-loop renormalization group equations, the masses of the new particles, the Higgs boson mass and, if applicable, the value of the gauge coupling at low energies for the new colour group are calculated.

2. Finite spectral triples

In this section, we will give the necessary basic definitions of almost-commutative geometries from a particle physics point of view. For our calculations, only the finite part matters, so we restrict ourselves to real, finite spectral triples in KO -dimension six: $(\mathcal{A}, \mathcal{H}, \mathcal{D}, J, \chi)$. Note that in the literature before [3–5], the finite part of the spectral triple was considered to be of KO -dimension zero. The change in this algebraic dimension amounts in some sign changes, i.e. the commutator for the real structure and the chirality changes into an anticommutator and the antiparticles have opposite chirality with respect to the particles.

2.1. Basic definitions

The algebra \mathcal{A} is a finite sum of matrix algebras $\mathcal{A} = \bigoplus_{i=1}^N M_{n_i}(\mathbb{K}_i)$, with $\mathbb{K}_i = \mathbb{R}, \mathbb{C}, \mathbb{H}$, where \mathbb{H} denotes the quaternions. A faithful representation ρ of \mathcal{A} is given on the finite-dimensional Hilbert space \mathcal{H} . The Dirac operator \mathcal{D} is a selfadjoint operator on \mathcal{H} and plays the role of the fermionic mass matrix. J is an anti-unitary involution, $J^2 = 1$, and is interpreted as the charge conjugation operator of particle physics. The chirality χ is a unitary involution, $\chi^2 = 1$, whose eigenstates with eigenvalue $+1(-1)$ are interpreted as right (left) particle states and $-1(+1)$ for right (left) antiparticle states. These operators are required to fulfil Connes’ axioms for spectral triples.

- $[J, \mathcal{D}] = \{J, \chi\} = \{\mathcal{D}, \chi\} = 0$,
 $[\chi, \rho(a)] = [\rho(a), J\rho(b)J^{-1}] = [[\mathcal{D}, \rho(a)], J\rho(b)J^{-1}] = 0, \quad \forall a, \quad b \in \mathcal{A}$.
- The chirality can be written as a finite sum $\chi = \sum_i \rho(a_i)J\rho(b_i)J^{-1}$. This condition is called *orientability*.
- The intersection form $\cap_{ij} := \text{tr}(\chi\rho(p_i)J\rho(p_j)J^{-1})$ is non-degenerate, $\det \cap \neq 0$. The p_i are minimal rank projections in \mathcal{A} . This condition is called *Poincaré duality*.

Now the Hilbert space \mathcal{H} and the representation are ρ decomposed into left and right, particle and antiparticle spinors and representations:

$$\mathcal{H} = \mathcal{H}_L \oplus \mathcal{H}_R \oplus \mathcal{H}_L^c \oplus \mathcal{H}_R^c, \quad \rho = \rho_L \oplus \rho_R \oplus \overline{\rho}_L^c \oplus \overline{\rho}_R^c.$$

In this representation, the Dirac operator has the form

$$\mathcal{D} = \begin{pmatrix} 0 & \mathcal{M} & 0 & 0 \\ \mathcal{M}^* & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{\mathcal{M}} \\ 0 & 0 & \overline{\mathcal{M}}^* & 0 \end{pmatrix},$$

where \mathcal{M} is the fermionic mass matrix connecting the left-handed and the right-handed fermions.

Since the individual matrix algebras have only one fundamental representation for $\mathbb{K} = \mathbb{R}, \mathbb{H}$ and two for $\mathbb{K} = \mathbb{C}$ (the fundamental one and its complex conjugate), ρ may be written as a direct sum of these fundamental representations with multiplicities

$$\rho\left(\bigoplus_{i=1}^N a_i\right) := \left(\bigoplus_{i,j=1}^N a_i \otimes 1_{m_{ji}} \otimes 1_{(n_j)}\right) \oplus \left(\bigoplus_{i,j=1}^N 1_{(n_i)} \otimes 1_{m_{ji}} \otimes \overline{a_j}\right).$$

The first summand denotes the particle sector and the second the antiparticle sector. For the dimensions of the unity matrices, we have $(n) = n$ for $\mathbb{K} = \mathbb{R}, \mathbb{C}$ and $(n) = 2n$ for $\mathbb{K} = \mathbb{H}$ and the convention $1_0 = 0$. The multiplicities m_{ji} are non-negative integers. Acting with the real structure J on ρ permutes the main summands and complex conjugates them. It is also possible to write the chirality as a direct sum

$$\chi = \left(\bigoplus_{i,j=1}^N 1_{(n_i)} \otimes \chi_{ji} 1_{m_{ji}} \otimes 1_{(n_j)} \right) \oplus \left(\bigoplus_{i,j=1}^N 1_{(n_i)} \otimes (-\chi_{ji}) 1_{m_{ji}} \otimes 1_{(n_j)} \right),$$

where $\chi_{ji} = \pm 1$ according to the previous convention on left-(right-)handed spinors. One can now define the multiplicity matrix $\mu \in M_N(\mathbb{Z})$ such that $\mu_{ji} := \chi_{ji} m_{ji}$. This matrix is symmetric and decomposes into a particle and an antiparticle matrix, the second being just the particle matrix transposed, $\mu = \mu_P + \mu_A = \mu_P - \mu_P^T$. The intersection form of the Poincaré duality is now simply $\cap = \mu - \mu^T$, see [16]. Note that in contrast to the case of KO -dimension zero, the multiplicity matrix is now antisymmetric rather than symmetric.

2.2. Obtaining the Yang–Mills–Higgs theory

To construct the actual Yang–Mills–Higgs theory, one starts out with the fixed (for convenience flat) Dirac operator of a four-dimensional spacetime with a fixed fermionic mass matrix. To generate curvature, a general coordinate transformation is performed and then the Dirac operator is fluctuated. This can be achieved by lifting the automorphisms of the algebra to the Hilbert space, unitarily transforming the Dirac operator with the lifted automorphisms and then building linear combinations. Again it is sufficient to restrict the treatment to the finite case.

All the automorphisms of matrix algebras connected to the unity element, $\text{Aut}(\mathcal{A})^e$, are inner, i.e. they are of the form

$$i_u a = u a u^*, \quad a \in \mathcal{A}, \tag{2.1}$$

where

$$u \in \mathcal{U}(\mathcal{A}) = \{u \in \mathcal{A} \mid u^* u = u u^* = 1\} \tag{2.2}$$

is an element of the group of unitaries of the algebra and i is a map from the unitaries into the inner automorphisms $\text{Int}(\mathcal{A})$

$$\begin{aligned} i : \mathcal{U}(\mathcal{A}) &\longrightarrow \text{Int}(\mathcal{A}) \\ u &\longmapsto i_u. \end{aligned} \tag{2.3}$$

In the kernel of i are the central unitaries, which commute with all elements in \mathcal{A} . These inner automorphisms $\text{Int}(\mathcal{A})$ are equivalent to the group of unitaries $\mathcal{U}(\mathcal{A})$ modulo the central unitaries $\mathcal{U}^c(\mathcal{A})$.

The Abelian algebras \mathbb{R} and \mathbb{C} do not possess any inner automorphisms. Remarkably, the quaternions and the matrix algebras over the complex numbers produce the kind of inner automorphisms that correspond to the non-Abelian gauge groups of the standard model. Note that the exceptional groups do not appear. They are the automorphism groups of non-associative algebras.

As in the Riemannian case, the automorphisms close to the identity are going to be lifted to the Hilbert space. This lift has a simple closed form [17], $L = \hat{L} \circ i^{-1}$ with

$$\hat{L}(u) = \rho(u) J \rho(u) J^{-1}. \tag{2.4}$$

Here, two crucial problems occur. The symmetry group of the standard model contains an Abelian sub-group $U(1)_Y$. But the inner automorphisms do not contain any Abelian sub-groups by definition. Furthermore, the lift is multivalued for matrix algebras over the complex

numbers since the kernel of i contains an $U(1)$ -group. Note that neither the matrix algebras over the reals nor those over the quaternions have any central unitaries close to the identity. The solution to both of these problems is to centrally extend the lift, i.e. to adjoin some central elements [15]. One has to distinguish between central unitaries stemming from the Abelian algebra \mathbb{C} and those from the non-Abelian matrix algebras $M_n(\mathbb{C})$, $n \geq 2$. To simplify, let the algebra \mathcal{A} be a sum of matrix algebras over the complex numbers. Furthermore, the commutative and noncommutative sub-algebras will be separated,

$$\mathcal{A} = \mathbb{C}^M \oplus \bigoplus_{k=1}^N M_{n_k}(\mathbb{C}) \ni (b_1, \dots, b_M, c_1, \dots, c_N), \quad n_k \geq 2. \tag{2.5}$$

The group of unitaries $\mathcal{U}(\mathcal{A})$ and the group of central unitaries $\mathcal{U}^c(\mathcal{A})$ are then given by

$$\begin{aligned} \mathcal{U}(\mathcal{A}_f) &= U(1)^M \times U(n_1) \times \dots \times U(n_N) \ni u = (v_1, \dots, v_M, w_1, \dots, w_N), \\ \mathcal{U}^c(\mathcal{A}_f) &= U(1)^{M+N} \ni u^c = (v_1, \dots, v_M, w_1^c 1_{n_1}, \dots, w_N^c 1_{n_N}). \end{aligned} \tag{2.6}$$

The inner automorphisms follows

$$\text{Int}(\mathcal{A}) = \mathcal{U}(\mathcal{A})/\mathcal{U}^c(\mathcal{A}) \ni u^{in} = (1, \dots, 1, w_1^{in}, \dots, w_N^{in}), \tag{2.7}$$

with $w_j^{in} \in \mathcal{U}(M_{n_j})/U(1)$. The lift $L = \hat{L} \circ i^{-1}$ can be written explicitly with

$$\hat{L} = \rho(1, \dots, 1, w_1, \dots, w_M) J \rho(\dots) J^{-1}. \tag{2.8}$$

It is multivalued due to the kernel of i , $\ker(i) = \mathcal{U}^c(\mathcal{A}_f)$. This multivaluedness can be cured by introducing an additional lift ℓ for the central unitaries, which is restricted to those unitaries $\mathcal{U}^{nc}(\mathcal{A})$ stemming from the noncommutative part of the algebra,

$$\begin{aligned} \ell(w_1^c, \dots, w_N^c) &:= \rho \left(\prod_{j_1=1}^N (w_{j_1}^c)^{q^{1,j_1}}, \dots, \prod_{j_M=1}^N (w_{j_M}^c)^{q^{M,j_M}}, \prod_{j_{M+1}=1}^N (w_{j_{M+1}}^c)^{q^{1,j_{M+1}}} 1_{n_1}, \dots, \right. \\ &\quad \left. \times \prod_{j_{M+N}=1}^N (w_{j_{M+N}}^c)^{q^{1,j_{M+N}}} 1_{n_N} \right) J \rho(\dots) J^{-1}, \end{aligned} \tag{2.9}$$

with the $(M + N) \times N$ matrix of charges $q_{k,j}$. The extended lift \mathbb{L} is then defined as

$$\mathbb{L}(u^i, w^c) := (\hat{L} \circ i^{-1})(u^i) \ell(w^c), \quad u^i \in \text{Int}(\mathcal{A}), \quad w^c \in \mathcal{U}^{nc}(\mathcal{A}).$$

For convenience, this lift will be written $\mathbb{L}(u)$ without making the specific distinction between the unitaries and the central unitaries.

In this way, the Abelian gauge groups have been introduced and the multivaluedness has been reduced, depending on the choice of the matrix of charges.

The fluctuation ${}^f\mathcal{D}$ of the Dirac operator \mathcal{D} is given by a finite collection f of real numbers r_j and algebra automorphisms $u_j \in \text{Aut}(\mathcal{A})^e$ such that

$${}^f\mathcal{D} := \sum_j r_j \mathbb{L}(u_j) \mathcal{D} \mathbb{L}(u_j)^{-1}, \quad r_j \in \mathbb{R}, \quad u_j \in \text{Aut}(\mathcal{A})^e.$$

These fluctuated Dirac operators build an affine space which serves as the configuration space for the Yang–Mills–Higgs theory. Only fluctuations with real coefficients are considered since ${}^f\mathcal{D}$ must remain selfadjoint. The sub-matrix of the fluctuated Dirac operator ${}^f\mathcal{D}$ which is equivalent to the mass matrix \mathcal{M} , is often denoted by φ , the ‘Higgs scalar’, in physics literature.

As mentioned in the introduction, an almost-commutative geometry is the tensor product of a finite noncommutative triple with an infinite, commutative spectral triple. By Connes’

reconstruction theorem [18, 19], it is known that the latter comes from a Riemannian spin manifold, which will be taken to be any four-dimensional, compact manifold. The spectral action of this almost-commutative spectral triple is defined to be the number of eigenvalues of the Dirac operator ${}^f\mathcal{D}$ up to a cutoff Λ . Via the heat-kernel expansion one finds, after a long and laborious calculation [2, 5], a Yang–Mills–Higgs action combined with the Einstein–Hilbert action, a cosmological constant, a term containing the Weyl tensor squared as well as a conformal coupling of the Higgs field to the curvature scalar:

$$\begin{aligned}
 S_{CC}[e, A_{L/R}, \varphi] &= \text{tr} \left[h \left(\frac{{}^f\mathcal{D}^2}{\Lambda^2} \right) \right] \\
 &= \int_M \left\{ \frac{2\Lambda_c}{16\pi G} - \frac{1}{16\pi G} R + a(5R^2 - 8R_{\mu\nu}R^{\mu\nu} - 7R_{\mu\nu\lambda\tau}R^{\mu\nu\lambda\tau}) \right. \\
 &\quad + \sum_i \frac{1}{2g_i^2} \text{tr} F_{i\mu\nu}^* F_i^{\mu\nu} + \frac{1}{2} (D_\mu \varphi)^* D^\mu \varphi + \lambda \text{tr}(\varphi^* \varphi)^2 - \frac{1}{2} \mu^2 \text{tr}(\varphi^* \varphi) \\
 &\quad \left. + \frac{1}{12} \text{tr}(\varphi^* \varphi) R \right\} dV + \mathcal{O}(\Lambda^{-2}), \tag{2.11}
 \end{aligned}$$

where $h : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a positive test function. The coupling constants are functions of the first moments h_0, h_2 and h_4 of the test function

$$\begin{aligned}
 \Lambda_c &= \alpha_1 \frac{h_0}{h_2} \Lambda^2, & G &= \alpha_2 \frac{1}{h_2} \Lambda^{-2}, & a &= \alpha_3 h_4, \\
 g_i^2 &= \alpha_{4i} \frac{1}{h_4}, & \lambda &= \alpha_5 \frac{1}{h_4}, & \mu^2 &= \alpha_5 \frac{h_2}{h_4} \Lambda^2.
 \end{aligned} \tag{2.12}$$

The curvature terms $F_{\mu\nu}$ and the covariant derivative D_μ are in the standard form of the Yang–Mills–Higgs theory. The constants α_j depend in general on the special choice of matrix algebra and on the Hilbert space, i.e. on the particle content. For details of the computation, we refer to [2, 5].

This action is valid at the cutoff Λ where it ties together the coupling constants g_i of the gauge connections and the Higgs coupling λ since they originate from the same heat-kernel coefficient. For the standard model with three generations, the calculation of the gauge couplings in (2.12) imposes at Λ conditions on the $U(1)_Y, SU(2)_w$ and $SU(3)_c$ couplings g_1, g_2 and g_3 comparable to those of grand unified theories:

$$5g_1^2 = 3g_2^2 = 3g_3^2. \tag{2.13}$$

But since the lift of the automorphisms produces extra free parameters through the $U(1)$ central charges, the first equality can always be fulfilled by a different choice of the central charge. Therefore, only the gauge couplings of noncommutative gauge groups underlie constraints from the spectral action.

In the same way as for the gauge couplings, the spectral action also implies constraints for the quartic Higgs coupling λ and the Yukawa couplings. The full set of constraints for the standard model reads [3, 5, 7]

$$3g_2^2 = 3g_3^2 = 3 \frac{Y_2^2}{H} \frac{\lambda}{24} = \frac{3}{4} Y_2. \tag{2.14}$$

Here, Y_2 is the sum of all Yukawa couplings g_f squared, H is the sum of all Yukawa couplings raised to the fourth power. Our normalizations are $m_f = \sqrt{2}(g_f/g_2)m_W, (1/2)(\partial\varphi)^2 + (\lambda/24)\varphi^4$.

As we will see in the following, the grand unified constraint $g_2^2 = g_3^2$ at the cutoff Λ is a very special case. It is valid for the standard model but in general it will not hold. The model

presented in this paper is one example for different constraints for g_2 and g_3 at the cutoff energy. For possible extensions of the standard model within the framework of almost-commutative geometry, these constraints may limit the particle content in a crucial way.

3. The spectral triple

The basic entity of the spectral triple is the matrix algebra. For the model under consideration, it is

$$\mathcal{A} = \mathbb{H} \oplus \mathbb{C} \oplus M_3(\mathbb{C}) \oplus \mathbb{C} \oplus M_D(\mathbb{C}) \oplus \mathbb{C} \ni (a, b, c, d, e, f). \tag{3.1}$$

It has the algebra \mathcal{A}_{SM} of the standard model as a sub-algebra and contains two new summands \mathcal{A}_{new} .

$$\mathcal{A}_{SM} = \mathbb{H} \oplus \mathbb{C} \oplus M_3(\mathbb{C}) \oplus \mathbb{C} \quad \text{and} \quad \mathcal{A}_{new} = M_D(\mathbb{C}) \oplus \mathbb{C}. \tag{3.2}$$

This particular model is inspired by the classification of almost-commutative geometries presented in [13]. For one generation in the standard model and the new particles, it is a slightly modified irreducible spectral triple in the sense that one right-handed new particle can be deleted from the spectral triple without violating the axioms. But in this case the physical model would not be free of anomalies and has therefore to be discarded. We do not want to go into all the details of the construction, but for the interested reader we give the Krajewski diagram of this almost-commutative spectral triple in the appendix.

The representation of the algebra on the Hilbert space is the usual one for the standard model. For the new part, the representation is given by

$$\begin{aligned} \rho_L(a) &= a \otimes 1_D, & \rho_R(f) &= \begin{pmatrix} f 1_D & 0 \\ 0 & \bar{f} 1_D \end{pmatrix}, \\ \rho_L^c(e) &= 1_2 \otimes e, & \rho_R^c(e) &= \begin{pmatrix} e & 0 \\ 0 & e \end{pmatrix}. \end{aligned} \tag{3.3}$$

The complete representation is then the direct sum of the standard model representation and the new part:

$$\rho = \rho_{SM} \oplus \rho_{new} \quad \text{with} \quad \rho_{new}(a, e, f) = \rho_L(a) \oplus \rho_R(f) \oplus \rho_L^c(e) \oplus \rho_R^c(e). \tag{3.4}$$

The same holds for the Hilbert space, $\mathcal{H} = \mathcal{H}_{SM} \oplus \mathcal{H}_{new}$. For one generation of the new particles, the Hilbert space is

$$\mathcal{H}_{new} = (\mathbb{C}^2 \otimes \mathbb{C}^D) \oplus (\mathbb{C} \otimes \mathbb{C}^D) \oplus (\mathbb{C} \otimes \mathbb{C}^D) \oplus \text{antiparticles}. \tag{3.5}$$

The dimension of \mathcal{H}_{new} depends on the number of generations N of the new particles and the size D of the sub-algebra $M_D(\mathbb{C})$ and reads $\dim(\mathcal{H}_{new}) = 8ND$.

We will denote the spinors of the new particles by ψ_1 and ψ_2 . They appear as left-handed $SU(2)$ doublets and right-handed $SU(2)$ singlets

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}_L \oplus (\psi_1)_R \oplus (\psi_2)_R \oplus \begin{pmatrix} \psi_1^c \\ \psi_2^c \end{pmatrix}_L \oplus (\psi_1^c)_R \oplus (\psi_2^c)_R \in \mathcal{H}_{new}. \tag{3.6}$$

Furthermore, every ψ_i is a $SU(D)$ D-plet for $D \geq 2$. The Dirac operator contains the Yukawa couplings of the new particles

$$\mathcal{M}_{new} = \begin{pmatrix} g_{\psi_1} & 0 \\ 0 & g_{\psi_2} \end{pmatrix} \otimes 1_D \quad \text{with} \quad g_{\psi_1}, g_{\psi_2} \in \mathbb{C} \tag{3.7}$$

and is given by

$$D_{\text{new}} = \begin{pmatrix} 0 & \mathcal{M}_{\text{new}} & 0 & 0 \\ \mathcal{M}_{\text{new}}^* & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{\mathcal{M}_{\text{new}}} \\ 0 & 0 & \overline{\mathcal{M}_{\text{new}}}^* & 0 \end{pmatrix}. \tag{3.8}$$

For more than one generation, it is of course possible to introduce a CKM-type matrix which mixes the generations. But to keep the analysis of the model as simple as possible, we will not include family mixing.

From the Krajewski diagram, figure A1 in the appendix, it is straightforward to see that all the axioms for the spectral triple are fulfilled. To the three generations of the standard model, one may add any number N of generations of the new particles with an arbitrarily large sub-algebra $M_D(\mathbb{C})$. In the following, we will investigate the physical models with respect to the number of generations of new particles N and with respect to the size D of the sub-algebra.

4. The gauge group, the lift and the constraints

The group of unitaries of the noncommutative part of the algebra is

$$\mathcal{U}^{nc}(\mathcal{A}) = SU(2)_w \times U(3) \times U(D) \ni (v, w, s) \quad \text{for} \quad D \geq 2. \tag{4.1}$$

In the case of $D = 1$, the group is just $\mathcal{U}^{nc}(\mathcal{A}) = SU(2) \times U(3)$, as for the standard model. Define $u := \det(w) \in U(1)_1$ and $r := \det(s) \in U(1)_2$. Note that in case of $D = 1$, s and r can simply be dropped from the following calculations.

The lift of the unitaries decomposes into a standard model part and a part for the new particles,

$$\begin{aligned} \mathbb{L}(v, u^{p_1} s^{q_1}, u^{p_2} s^{q_2} w, u^{p_3} s^{q_3}, u^{p_4} s^{q_4} r, u^{p_5} s^{q_5}) &= \mathbb{L}_{\text{SM}}(v, u^{p_1} s^{q_1}, u^{p_2} s^{q_2} w, u^{p_3} s^{q_3}) \\ &\oplus \mathbb{L}_{\text{new}}(v, u^{p_4} s^{q_4} r, u^{p_5} s^{q_5}), \end{aligned} \tag{4.2}$$

with $p_i, q_i \in \mathbb{Z}$. This lift produces *a priori* two $U(1)$ groups through the central extensions u and r . But it has been shown in [13] that in the case of two $U(1)$ groups the requirement of being anomaly free results in proportional couplings of the $U(1)$'s to the standard model particles. The two photons can therefore be linearly combined into a physical photon and an unphysical photon that does not couple to the standard model. Without loss of generality, we can therefore set $q_1 = q_2 = q_3 = 0$. For the standard model part of the lift, one finds then $p_1 = -p_3 = -1/2$ and $p_2 = 1/6 - 1/3$ from anomaly cancellation. This reduces $U(3)$ to $U(1)_Y \times SU(3)_c$ in the correct representation.

The exact form of the lift for the new particles is given by

$$\mathbb{L}_{\text{new}}(v, u^{p_4} s^{q_4} r, u^{p_5} s^{q_5}) = \text{diag}[u^{p_4} s^{q_4} v \otimes r; u^{p_4+p_5} s^{q_4+q_5} r, u^{p_4-p_5} s^{q_4-q_5} r]. \tag{4.3}$$

Being anomaly free and requiring that the corresponding left-handed and right-handed particles are equally charged under the little group leads to $p_4 = q_5 = 0, q_4 = -1/D$ and $p_5 = p_1 = -1/2$. Since the normalization of the lift for the right-handed electron is chosen to be $Y_{e_R} = 2p_1 = -1$, one sees immediately that $Y_{\psi_L} = 0, Y_{\psi_R} = \pm p_1 = \mp 1/2$. Therefore, the electromagnetic charge of the new particles is $Q_{\text{el}} = \pm 1/2e$. This charge

assignment is summarized in the following table:

	I	I_3	Y_{new}	Q_{el}	$SU(D)$
$(\psi_1)_L$	2	$+\frac{1}{2}$	0	$+\frac{e}{2}$	D
$(\psi_2)_L$	2	$-\frac{1}{2}$	0	$-\frac{e}{2}$	D
$(\psi_1)_R$	1	0	$\frac{1}{2}$	$+\frac{e}{2}$	D
$(\psi_2)_R$	1	0	$-\frac{1}{2}$	$-\frac{e}{2}$	D

Plugging in the numbers one finds for lift

$$\begin{aligned} \mathbb{L}_{\text{new}}(v, s^{-1/D}r, u^{-1/2}) &= \text{diag}[s^{-1/D}v \otimes r; u^{-1/2}s^{-1/D}r, u^{1/2}s^{-1/D}r] \\ &= \text{diag}[v \otimes \tilde{r}; u^{-1/2}\tilde{r}, u^{1/2}\tilde{r}], \end{aligned} \tag{4.4}$$

with $\tilde{r} \in SU(D)_{\text{new}}$. For $D \geq 2$ the gauge group of the model is then

$$U(1)_Y \times SU(2)_w \times SU(3)_c \times SU(D)_{\text{new}} \tag{4.5}$$

and for $D = 1$ it is just the standard model gauge group. It is also remarkable that the inner fluctuations of the mass matrix \mathcal{M}_{new} with the lift \mathbb{L}_{new} produce exactly the standard model Higgs field

$$\sum_j r_j \mathbb{L}_{L,\text{new}}(v_i, \tilde{r}_i, u_i^{-1/2}) \mathcal{M}_{\text{new}} \mathbb{L}_{R,\text{new}}^{-1}(v_i, \tilde{r}_i, u_i^{-1/2}) = \varphi_{\text{SM}} \mathcal{M}_{\text{new}}, \tag{4.6}$$

where the subscripts L and R indicate the left-handed and the right-handed parts of the lift. Therefore, \mathcal{M}_{new} contains the Yukawa couplings of the new model, in exact analogy to the standard model.

From the spectral action, one obtains now immediately the Lagrangian for the new particles,

$$\begin{aligned} \mathcal{L}_{\text{new}} &= -\frac{1}{4} \text{tr}(G_{\mu\nu} G^{\mu\nu}) + i \sum_{i=1,\dots,N} (\bar{\psi}_1, \bar{\psi}_2)_L^i D_L^\psi \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}_L^i \\ &+ i \sum_{i=1,\dots,N} (\bar{\psi}_1)_R^i D_R^{\psi_1} (\psi_1)_R^i + i \sum_{i=1,\dots,N} (\bar{\psi}_2)_R^i D_R^{\psi_2} (\psi_2)_R^i \\ &- \sum_{i=1,\dots,N} (g_{\psi_1})_i (\bar{\psi}_1, \bar{\psi}_2)_L^i \varphi_{\text{SM}} (\psi_1)_R^i - \sum_{i=1,\dots,N} (g_{\psi_2})_i (\bar{\psi}_1, \bar{\psi}_2)_L^i \varphi_{\text{SM}} (\psi_2)_R^i \\ &+ \text{hermitian conjugate}, \end{aligned} \tag{4.7}$$

where the covariant derivatives are given by

$$D_L^\psi = \partial_\mu + ig_2 W_\mu^k \frac{\tau_k}{2} + ig_4 G_\mu^a t_a \tag{4.8}$$

$$D_R^{\psi_i} = \partial_\mu + ig_1 \frac{Y_i}{2} B_\mu + ig_4 G_\mu^a t_a. \tag{4.9}$$

Here, g_1 and g_2 are the standard model $U(1)_Y$ and $SU(2)_w$ gauge couplings. For $D \geq 2$, the $SU(D)$ gauge coupling is g_4 , t_a are the corresponding generators and G_μ^a are the gauge fields

with the usual curvature tensor

$$G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu - g_4[G_\mu, G_\nu]. \quad (4.10)$$

The $SU(D)$ terms have of course to be dropped from all equations if $D = 1$.

From the spectral action, it is now straightforward to calculate the constraints on the gauge couplings, the quartic Higgs coupling and the Yukawa couplings. The normalization of the quartic Higgs coupling is taken to be the same as for the standard model:

$$Ng_4^2 = 3g_3^2 = \left(3 + \frac{ND}{4}\right)g_2^2 = 3\frac{Y_2^2}{H}\frac{\lambda}{24} = \frac{3}{4}Y_2. \quad (4.11)$$

Y_2 and H include now the Yukawa couplings of the new particles in the standard way.

One notes immediately that models beyond the standard model in almost-commutative geometry will in general not exhibit the constraint $g_2 = g_3$ from grand unified theories. This is a qualitatively new feature and may prove to be important in restricting possible extensions of the standard model within the framework of the spectral action.

In the following analysis, we will for simplicity assume that all the Yukawa couplings of the new particles in all generations are equal, i.e. $g_{\psi_1} = g_{\psi_2} =: g_\psi$. For more realistic models, one would of course admit different Yukawa couplings, but as a first estimation of the particle masses equal couplings should be sufficient. Furthermore, we will assume three generations for the standard model particles and we will neglect all the standard model Yukawa couplings safe on the top quark coupling g_t . Under these assumptions, the constraints on the couplings at the cutoff Λ read:

$$g_3^2 = \left(1 + \frac{ND}{12}\right)g_2^2, \quad (4.12)$$

$$g_4^2 = \left(\frac{3}{N} + \frac{D}{4}\right)g_2^2, \quad (4.13)$$

$$g_t^2 = \frac{4 + \frac{ND}{3}}{3 + 2NDR^2}g_2^2 \quad \text{with} \quad R := \frac{g_\psi}{g_t}, \quad (4.14)$$

$$\lambda = 8\left(3 + \frac{ND}{4}\right)\frac{3 + 2NDR^4}{(3 + 2NDR^2)^2}g_2^2. \quad (4.15)$$

Here it becomes obvious that equation (4.12) no longer resembles the grand unification condition $g_3^2 = g_2^2$ as in the case of the pure standard model.

The strategy to find the masses of the new particles and the mass of the Higgs boson is now the following. With the help of the renormalization group equation for g_2 and g_3 , one determines the cutoff via condition (4.12). There one can fix g_4 and λ with conditions (4.13) and (4.15). The last free parameter is the ratio R of the Yukawa coupling g_ψ of the new particles and the top quark Yukawa coupling g_t . This ratio is fixed by the requirement that the renormalization group flow produces the measured pole mass of the top quark, $m_t = 170.9 \pm 2.6$ GeV [22].

5. The renormalization group equations

We will now give the one-loop β -functions of the standard model with three generations of the standard model particles, N generations of the new particles with either no new colour, i.e. $D = 1$ and $M_D(\mathbb{C}) = \mathbb{C}$, or with an $SU(D)$ colour group and $D \geq 2$. They will serve to evolve the constraints (4.11) from $E = \Lambda$ down to our energies $E = m_Z$. We set: $t := \ln(E/m_Z)$, $dg/dt =: \beta_g$ and $\kappa := (4\pi)^{-2}$. We will neglect all the standard model fermion masses below the top mass and also neglect threshold effects.

The β -functions are [20, 21]

$$\beta_{g_i} = \kappa b_i g_i^3, \quad b_i = \left(\frac{41}{6} + \frac{ND}{3}, -\frac{19}{6} + \frac{ND}{3}, -7, -\frac{11}{3}N + \frac{4}{3}D \right), \quad (5.1)$$

$$\beta_t = \kappa \left[-\sum_i c_i^t g_i^2 + Y_2 + \frac{3}{2}g_t^2 \right] g_t, \quad (5.2)$$

$$\beta_{\psi_1} = \kappa \left[-\sum_i c_i^\psi g_i^2 + Y_2 + \frac{3}{2}g_{\psi_1}^2 - \frac{3}{2}g_{\psi_2}^2 \right] g_{\psi_1}, \quad (5.3)$$

$$\beta_{\psi_2} = \kappa \left[-\sum_i c_i^\psi g_i^2 + Y_2 + \frac{3}{2}g_{\psi_2}^2 - \frac{3}{2}g_{\psi_1}^2 \right] g_{\psi_2}, \quad (5.4)$$

$$\beta_\lambda = \kappa \left[\frac{9}{4} (g_1^4 + 2g_1^2 g_2^2 + 3g_2^4) - (3g_1^2 + 9g_2^2) \lambda + 4Y_2 \lambda - 12H + 4\lambda^2 \right], \quad (5.5)$$

with

$$c_i^t = \left(\frac{17}{12}, \frac{9}{4}, 8 \right), \quad c_i^\psi = \left(\frac{3}{4}, \frac{9}{4}, 0, 3 \frac{D^2 - 1}{D} \right), \quad (5.6)$$

$$Y_2 = 3g_t^2 + NDg_{\psi_1}^2 + NDg_{\psi_2}^2, \quad H = 3g_t^4 + NDg_{\psi_1}^4 + NDg_{\psi_2}^4. \quad (5.7)$$

For the case $D = 1$, the β -functions of g_4 and c_4^ψ are to be ignored. The gauge couplings decouple from the other equations and have identical evolutions in both energy domains:

$$g_i(t) = g_{i0} / \sqrt{1 - 2\kappa b_i g_{i0}^2 t}. \quad (5.8)$$

The initial conditions are taken from experiment [22]

$$g_{10} = 0.3575, \quad g_{20} = 0.6514, \quad g_{30} = 1.221. \quad (5.9)$$

Then, the unification scale Λ is the solution of $(1 + \frac{ND}{12}) g_2(\ln(\Lambda/m_Z)) = g_3(\ln(\Lambda/m_Z))$,

$$\Lambda = m_Z \exp \frac{g_{20}^{-2} - (1 + \frac{ND}{12})^2 g_{30}^{-2}}{2\kappa (b_2 - (1 + \frac{ND}{12})^2 b_3)}, \quad (5.10)$$

and depends on the number of generations of new particles N and the size of the matrix algebra D .

6. The masses and the couplings at m_Z

We require that all couplings remain perturbative and we obtain the pole masses of the Higgs, the top and the new particles:

$$m_H^2 = \frac{4}{3} \frac{\lambda(m_H)}{g_2(m_Z)^2} m_W^2, \quad m_t = \sqrt{2} \frac{g_t(m_t)}{g_2(m_t)} m_W, \quad m_{\psi_1} = m_{\psi_2} = \frac{g_\psi(m_\psi)}{g_2(m_Z)^2} m_W^2. \quad (6.1)$$

As an experimental input, we have the initial conditions of the three standard model gauge couplings (5.9) and the mass of the top quark, $m_t = 170.9 \pm 2.6$ GeV [22]. As mentioned before, the masses of the new particles are assumed to be equal. With the constraints (4.12)–(4.15), we can now determine their numerical value via the renormalization group flow and we can also determine the mass of the Higgs boson for the respective model.

Let us start with the case $D = 1$, $M_D(\mathbb{C}) = \mathbb{C}$. The gauge group for this model is just the standard model gauge group. For up to three generations of the new particles, the resulting masses and cutoff energies are summarized in the following table:

N	Λ (GeV)	m_H (GeV)	m_ψ (GeV)
1	5.3×10^{13}	167.3 ± 3.4	69.3 ∓ 3.5
2	3.0×10^{11}	172.0 ± 3.2	53.7 ∓ 2.5
3	7.4×10^9	177.8 ± 2.5	48.0 ∓ 1.6

The new particles are very light. And since they possess electromagnetic charge, they should clearly have been detected if they existed [22]. This model has therefore to be discarded.

Next we consider the case $D = 2$. The gauge group for the model is $U(1)_Y \times SU(2)_w \times SU(3)_c \times SU(2)_{\text{new}}$. The standard model particles and the Higgs boson are $SU(2)_{\text{new}}$ singlets. For one generation, it is not possible to solve the constraint (4.13) within the real numbers, i.e. $g_4(t)$ has a pole below the cutoff. For two and three generations, the resulting masses and cutoff energies are

N	Λ (GeV)	m_H (GeV)	m_ψ (GeV)	$g_4(m_Z)$
2	4.4×10^8	182.3 ± 2.3	69.6 ∓ 2.3	1.50
3	8.3×10^6	196.7 ± 2.5	50.2 ∓ 1.7	0.91

The detectability of these particles is not as obvious as in the case without a new colour. First of all, the gauge coupling g_4 is strong, so one should expect confinement. Therefore, the new particles will, as quarks, not appear as free particles but bound into colour singlets. These composite particles could allow to escape from detectors if they are neutral and thus hide the new particles from detection. It is beyond the scope of this paper to give an analysis of the phenomenological details of the models, so we will postpone this analysis for a later publication.

For $D \geq 2$ the gauge coupling $g_4(t)$ has a pole below the cutoff for one and two generations of the new particles. So three generations is the minimal number. We give the cutoff energies, the masses of the new particles and the Higgs mass with respect to D are for three generations of the new particles.

D	Λ (GeV)	m_H (GeV)	m_ψ (GeV)	$g_4(m_Z)$
3	2.1×10^5	217.0 ± 1.5	56.5 ∓ 1.1	1.22
4	2.0×10^4	241.2 ± 1.6	62.3 ∓ 1.3	1.54
5	4.1×10^3	268.3 ± 0.8	65.8 ∓ 0.7	1.78
6	1.3×10^3	300.6 ± 0.8	63.6 ∓ 0.7	1.81
7	524	338.2 ± 0.7	57.6 ∓ 0.4	1.70
8	261	379.3 ± 0.9	50.7 ∓ 0.5	1.53

With respect to the detectability, the same arguments apply as for the case $D = 2$. The coupling g_4 is very strong for all models, so small confinement radii are to be expected. It is also interesting to note that the mass of the Higgs boson is strongly dependent on the cutoff scale. This reminds us of the older Connes–Lott model [23] which predicted a Higgs mass of $m_{H,C-L} \sim 250\text{--}324$ GeV [24]. For $D \geq 8$ the cutoff energy becomes smaller than the Higgs mass. This is also an interesting new feature that deserves further investigation.

7. Conclusions and outlook

We have presented a particle model based on an almost-commutative geometry which contains the standard model as a sub-model. It provides an extension of the standard model with N generations of the new particles. These particles come as a left-handed $SU(2)_w$ doublet and two right-handed singlets. The requirement of being anomaly free forces them to have opposite electromagnetic charges with an absolute value of half the electron charge. Furthermore, the model allows these particles to have a new $SU(D)_{\text{new}}$ colour. In this case, they are equivalent to Okun’s θ -particles [14].

The spectral action puts strong constraints on the gauge couplings, the quartic Higgs coupling and the Yukawa couplings of this model at the cutoff scale. Using the standard renormalization group equations, these constraints allow us to calculate the masses of the new particles and the mass of the Higgs boson at low energies. The masses of the new particles, under the assumption of equal masses in all families, range then from ~ 48 GeV to ~ 69 GeV, where up to three generations and new colour groups up to $SU(8)_{\text{new}}$ have been considered. For the Higgs boson, the masses range from ~ 164 GeV to ~ 334 GeV.

One should note that these new particles could be the preferred decay products of the Higgs boson. If they can hide from direct detection, this would mean that the Higgs boson could also be much more difficult to detect.

It is also interesting to note that the cutoff scale of the spectral action is considerably lowered by the presence of these new particles. For three generations and eight colours, it sinks even below the Higgs mass, $\Lambda \sim 261$ GeV for a Higgs mass of $m_H \sim 334$ GeV. Remarkably, for colour groups larger than $SU(2)_{\text{new}}$, one has to add at least three generations of the new particles.

Many questions have not been considered in this article. Let us list some of these.

- Are the new particles directly detectable by existing experiments such as LEP and Tevatron?
- Will the new particles be detectable by future experiments such as LHC?
- What is the phenomenology of the model? What are, for example, the stable colour singlet states and confinement radii?
- Does the model contain a viable dark matter candidate?

This list is certainly not exhaustive and other interesting questions may arise. But the model shows clearly that how difficult the extension of the standard model within almost-commutative geometry will be in general. The constraints stemming from the spectral action together with the geometrical constraints from the spectral triple formalism restrict model building severely. Apart from the older AC-model [11, 12], which possesses a viable dark matter candidate, this is so far the only model which could be in concordance with the experiment.

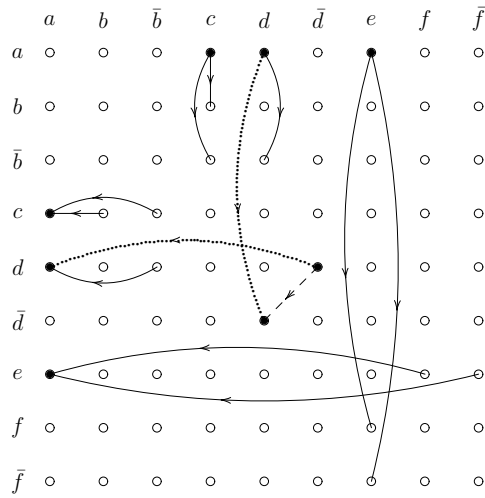


Figure A1. Krajewski diagram of the standard model with right-handed neutrino and Majorana-mass term depicted by the dashed arrow. The new particles reside in the e -line and the e -column.

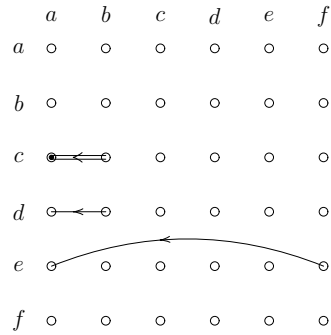


Figure A2. Minimal Krajewski diagram associated with the diagram in figure A1.

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Appendix. The Krajewski diagram

In this appendix, we present the Krajewski diagrams which were used to construct the model treated in this publication. Krajewski diagrams do for spectral triples what the Dynkin and weight diagrams do for groups and representations. For an introduction to the formalism of Krajewski we refer to [16, 13]. The Krajewski diagram for the model presented in this paper is depicted in figure A1. It shows one generation of the standard model particles and one generation of the new particles.

The arrows encoding the new particles are drawn on the e -line and the e -column. Note the similarity to the standard model quark sector which sits on the c -line and the c -column. The dotted arrows denote the possible right-handed neutrinos and the dashed arrow represents a possible Majorana mass term.

This diagram originates from the minimal diagram shown in figure A2. One remarks immediately that the right-handed neutrinos as well as one of the right-handed new particles can be neglected from the purely geometric point of view. But this model is not anomaly free and has therefore to be excluded.

The multiplicity matrix μ associated with the Krajewski diagram in figure A1, with three generations of the standard model particles and N generations of the new particles, can be directly read off to be

$$\mu = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -3 & 6 & 0 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 & 0 & 0 \\ -N & 0 & 0 & 0 & 0 & 2N \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (\text{A.1})$$

The axiom of the Poincaré duality is fulfilled since $\det(\mu - \mu^t) = 81(2N)^2 \neq 0$ for all $N \in \mathbb{N}$. Only the right-handed neutrinos violate the axiom of orientability [25] which is also the case for the pure standard model.

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